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The structure of the outer layers of the stars.

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1. Introduction. The theory of the inner structure of the stars, which is based on the hypothesis of radiative equilibrium, at present is not yet able to solve the problems imposed, i. e. to state the functional dependence between the fundamental physical, dynamic and geometric characteristics for every inner point of the stars. The reason of this circumstance is to be sought in our ignorance how the sources of energy in the stars are distributed. In fact, the unknown function of the optical mass τ namely $B(\tau)$, which represents the radiation of an absolutely black body at a temperature of the given layer, is determined by the following integral equation[1]):

$$f(\tau) = B(\tau) - \tfrac{1}{2}\int_0^\infty Ei\,|\tau - t|\,B(t)\,dt \tag{1}$$

where $f(\tau)$ is the function which represents the dependence of the quantity of energy formed, per every unit of the optical mass, on the optical mass itself, and where $Ei\,x = \int_1^\infty e^{-xt}/t\cdot dt$.

If we knew $f(\tau)$, the function $B(\tau)$ thereby would be determined as one-valued[2]) and the main part of the problem would be solved. However in the present time the physics cannot give us any notion about the form of the function $f(\tau)$, and therefore we have to determine it by empiric ways. As yet this empirical determination is stated only for the outer layers of the sun, proceeding from the distribution of the brightness over its disk[3]). However, for comprehensible reasons, this method is not applicable to stars.

In the present work we suggest a method of investigation of the outer layers of the stars which is based on the distribution of energy in their spectra. This problem as we are about to show, will come to the solution of an integral equation of the first type, the same solution we obtain in the form of a series of polynomes. In order to solve the above equation we have to make use of the tables appended to the present paper. Through this solution we are able to find the function $B(\tau)$ and, in case of need, the function $f(\tau)$ by means of one quadrature according to formula (1).

2. In our derivation we shall admit the coefficient of absorption to be independent of the length of the wave, i. e. we shall admit that the stars radiate and absorb like a »grey body«[4]). The visible brightness of a point, which is at an angular distance θ from the centre of the disk, will be expressed by the following formula:

$$I(\lambda, \theta) = \int_0^\infty E(\lambda, T)\, e^{-\int_0^h \varkappa\varrho\, dh\, \sec\theta}\, \sec\theta\, \varkappa\varrho\, dh \tag{2}$$

where $E(\lambda, T)$ is the radiation of an absolutely black body, the temperature being T and the length of the wave λ, h is the depth on which the layer is, ϱ the density and $\varkappa$ the coefficient of absorption. Denoting $\int_0^h \varkappa\varrho\, dh = \tau$ we have:

$$I(\lambda, \theta) = \int_0^\infty E(\lambda, T)\, e^{-\tau\sec\theta}\, \sec\theta\, d\tau\,. \tag{3}$$

In order to obtain the brightness of the whole star in the monochromatic light, the length of the wave being λ, we shall have to integrate $I(\lambda, \theta)$ for the whole disk. If we denote by r the distance of a point of the disk to the centre of the disk, the full brightness in the length of the wave will be expressed as follows:

$$2\pi\int_0^R r\,dr \int_0^\infty E(\lambda, T)\, e^{-\tau\sec\theta}\, \sec\theta\, d\tau$$

where R is the radius of the disk. For the unit of area we get:

$$\varphi(\lambda) = 2/R^2\cdot\int_0^R r\,dr\int_0^\infty E(\lambda, T)\, e^{-\tau\sec\theta}\,\sec\theta\, d\tau\,. \tag{4}$$

Thus $\varphi(\lambda)$ will represent, leaving aside a constant multiplier, the observed distribution of energy in the spectrum of a star. Altering the order of integration in (4) and noticing that between θ and r the following dependence exists:

$$\sec\theta = R/\sqrt{(R^2 - r^2)} \tag{5}$$

we find:

$$\varphi(\lambda) = 2/R\cdot\int_0^\infty E(\lambda, T)\, d\tau \int_0^R e^{-\tau R/\sqrt{(R^2 - r^2)}}\cdot r/\sqrt{(R^2 - r^2)}\cdot dr\,. \tag{6}$$

Supposing here:

$$x = \tau R/\sqrt{(R^2 - r^2)}$$

we get:

$$\varphi(\lambda) = 2\int_0^\infty E(\lambda, T)\,\tau\int_\tau^\infty e^{-x}/x^2\cdot dx\, d\tau\,. \tag{7}$$

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1) The derivation of this equation see AN **229**.89. The solution of this equation is represented in the form of a *Neumann* series. On the convergence of the same series see MN **87**.651–655.

2) On the uniqueness of the solution, i. e. the absence of any solutions being different from zero of the equation: $B(\tau) = \frac{1}{2}\int_0^\infty Ei\,|\tau - t|\,B(t)\,dt$ see MN **87**.209–215.

3) *Milne*; Phil. Trans. A **220**.217. *V. Parchomenko*; AN **227**.305. *N. Kosirev* und *V. Ambarzumian*; AN **229**.85.

4) *E. A. Milne*; Phil. Trans. A **223**.201–255.

Denoting:

$$\int_1^\infty e^{-\tau y}/y^2 \cdot dy = \tau \int_\tau^\infty e^{-x}/x^2 \cdot dx = Ei_2 \tau \quad (8)$$

we reduce the equation (7) to:

$$\varphi(\lambda) = 2 \int_0^\infty Ei_2 \tau \, E(\lambda, T) \, d\tau \, . \quad (9)$$

Let us somewhat transform this equation. Introducing a new function:

$$u = Ei_3 \tau = \int_1^\infty e^{-\tau y}/y^3 \cdot dy \quad (10)$$

we notice that

$$du = -Ei_2 \tau \cdot d\tau \, . \quad (11)$$

Thus the equation (9) is reduced to the form:

$$\varphi(\lambda) = 2 \int_0^{1/2} E(\lambda, T) \, du \, . \quad (12)$$

Let us consider E to be a function of $c_2/T = s$, and not a function of T. Then making use of *Planck*'s formula we get:

$$\lambda^5 \varphi(\lambda)/2c_1 = \int_0^{s_0} [e^{s/\lambda} - 1]^{-1} \cdot du/ds \cdot ds \quad (13)$$

where c_1 and c_2 in *Planck*'s formula represent constant quantities, namely $c_1 = 3.696 \cdot 10^{-5}$ Erg cm^2 sec^{-1}, $c_2 = 1.435$ cm deg and $s_0 = c_2/T_0$, wherein T_0 is the temperature of the upper limit of a star. Let us denote the left hand side of equation (13) by $\psi(\lambda)$ and put $du/ds = K(s)$. In this case the equation (13) will be expressed as follows:

$$\psi(\lambda) = \int_0^\infty [e^{s/\lambda} - 1]^{-1} \cdot K(s) \, ds \, . \quad (14)$$

Thus we obtain an integral equation of the first type between the known function $\psi(\lambda)$ and the sought function $K(s)$.

3. Let us admit that the function $K(s)$ can be developed into a series of polynomes, which do not contain the zero powers.

$$K(s) = c_1 P_1(s) + c_2 P_2(s) + \cdots + c_n P_n(s) + \cdots \quad (15)$$

Substituting in (14) the above expression for $K(s)$ we find:

$$\psi(\lambda) = \int_0^\infty [e^{s/\lambda} - 1]^{-1} \sum_{i=1}^\infty c_i P_i(s) \, ds = \sum_{i=1}^\infty c_i \int_0^\infty [e^{s/\lambda} - 1]^{-1} \cdot P_i(s) \, ds \, . \quad (16)$$

If

$$P_i(s) = a_1^{(i)} s + a_2^{(i)} s^2 + \cdots + a_n^{(i)} s^n \quad (17)$$

then the integration (16) will come to the calculation of integrals having the form:

$$\int_0^\infty [e^{s/\lambda} - 1]^{-1} s^m \, ds \, .$$

The expression $[e^{s/\lambda} - 1]^{-1}$, as it is easy to convince oneself, can be developed into the series:

$$[e^{s/\lambda} - 1]^{-1} = e^{-s/\lambda} + e^{-2s/\lambda} + \cdots + e^{-hs/\lambda} + \cdots \quad (18)$$

consequently[1])

$$\int_0^\infty [e^{s/\lambda} - 1]^{-1} s^m \, ds = \sum_{h=1}^\infty \int_0^\infty e^{-hs/\lambda} s^m \, ds \, . \quad (19)$$

But we have:

$$\int_0^\infty e^{-hs/\lambda} s^m \, ds = \lambda^{m+1}/h^{m+1} \cdot \int_0^\infty e^{-x} x^m \, dx = \lambda^{m+1}/h^{m+1} \cdot \Gamma(m+1) \quad (20)$$

and thus

$$\int_0^\infty [e^{s/\lambda} - 1]^{-1} s^m \, ds = \lambda^{m+1} \Gamma(m+1) \sum_{h=1}^\infty 1/h^{m+1} = \lambda^{m+1} \Gamma(m+1) \, \zeta(m+1) \quad (21)$$

where *Riemann*'s notation is introduced:

$$\zeta(m+1) = \sum_{i=1}^\infty 1/i^{m+1} \, . \quad (22)$$

Likewise we find that:

$$Q_i(\lambda) = \int_0^\infty [e^{s/\lambda} - 1]^{-1} P_i(s) \, ds = a_1 \lambda^2 \Gamma(2) \zeta(2) + a_2 \lambda^3 \Gamma(3) \zeta(3) + \cdots + a_n \lambda^{n+1} \Gamma(n+1) \zeta(n+1) \quad (23)$$

and finally:

$$\psi(\lambda) = \sum_{i=1}^\infty c_i Q_i(\lambda) \, . \quad (24)$$

Quite in the same way, knowing the development of the function $\psi(\lambda)$ into polynomes $Q_i(\lambda)$ and making use of the formula (23) one can calculate from the coefficients of the same polynomes the corresponding coefficients of the polynome $P_i(s)$ connected with $Q_i(\lambda)$ by the equation:

$$Q_i(\lambda) = \int_0^\infty [e^{s/\lambda} - 1]^{-1} P_i(s) \, ds \quad (25)$$

and thus one can write the development of the function $K(s)$ already into polynomes $P_i(s)$. Still let us notice that if $Q_i(\lambda)$ contains merely even powers, $P_i(s)$ will contain merely odd ones, which fact is important for our further investigations.

4. We shall develop the function $\psi(\lambda)$ according to the system of orthogonal and normalised polynomes of the even power, to begin with the second one, in the interval of the alteration of λ from 0 to 1[2]). These polynomes may be obtained by means of orthogonalisation and normalisation of the following system of functions:

$$\lambda^2, \lambda^4, \lambda^6, \cdots \lambda^{2n}, \cdots \, . \quad (26)$$

In order to show the possibility of this development, it is

[1]) The possibility of the integration term by term, i. e. of formula (19) for $m \geq 1$ can be readily proved.
[2]) The scale for measuring λ will be selected afterwards.

indispensable to prove the completeness of the system of functions (26)[1].

For this purpose let us previously prove a theorem of general character:

Theorem: If we have a complete system of continuous and orthogonal functions, normalised in the interval (a, b):

$$\varphi_1(t),\ \varphi_2(t),\ \cdots,\ \varphi_n(t),\ \cdots \tag{27}$$

one can represent uniformly and approximatively any function $f(t)$ as a linear combination of the following functions:

$$\psi_1(t)=\int_a^t \varphi_1(t)\,dt,\ \psi_2(t)=\int_a^t \varphi_2(t)\,dt,\ \cdots,\ \psi_n(t)=\int_a^t \varphi_n(t)\,dt,\ \cdots \tag{28}$$

if $f(t)$ is equal to zero in case of $t=a$ and has a continuous derivative $f'(t)$.

In order to prove this theorem we shall have to make use of the following lemma:

Lemma: Suppose we have a sequence of continuous functions in the interval (a, b):

$$p_1(t),\ p_2(t),\ \cdots\ p_n(t),\ \cdots. \tag{29}$$

If we denote:

$$q_1(t)=\int_a^t p_1(t)\,dt \qquad q_2(t)=\int_a^t p_2(t)\,dt,\ \cdots$$

$$q_n(t)=\int_a^t p_n(t)\,dt \cdots \tag{30}$$

from the relation

$$\lim_{\nu\to\infty}\int_a^b p_\nu^2(t)\,dt=0 \tag{31}$$

follows that: $\quad q_\nu(t) \Rightarrow 0$

where $\Rightarrow$ represents a uniform convergence.

First of all let us notice that from the relation (31) follows:

$$\lim_{\nu\to\infty}\int_a^b q_\nu^2(t)\,dt=0. \tag{32}$$

In fact we have:

$$q_\nu^2(t)=\left(\int_a^t p_\nu(t)\,dt\right)^2 \leq (t-a)\int_a^t p_\nu^2(t)\,dt \leq (b-a)\,Np_\nu \tag{33}$$

as it follows from the *Booniakovsky-Schwarz* inequality, if we denote:

$$\int_a^b p_\nu^2(t)\,dt=Np_\nu. \tag{34}$$

From (33) follows:

$$\int_a^b q_\nu^2(t)\,dt \leq \int_a^b (b-a)\,Np_\nu\,dt=(b-a)^2\,Np_\nu. \tag{35}$$

From (31) and (35) follows (32). In order to prove the lemma it is easy to show that the sequence of functions $q_\nu(t)$ is uniformly continuous. We admit the contrary. In this case a value $\varepsilon>0$ exists such that no matter how little $\eta>0$ should be, it is possible to find such number ν_k, t_k and η_k, whereby $|\eta_k|<\eta$, that

$$|q_{\nu_k}(t_k+\eta_k)-q_{\nu_k}(t_k)|>\varepsilon \tag{36}$$

and moreover we shall have an infinite quantity of numbers ν_k. Let us write the inequality (36) as follows:

$$\left|\int_{t_k}^{t_k+\eta_k} p_{\nu_k}(t)\,dt\right|>\varepsilon.$$

In virtue of the *Booniakovsky-Schwarz* inequality we have:

$$\left(\int_{t_k}^{t_k+\eta_k} p_{\nu_k}(t)\,dt\right)^2 \leq \eta_k\int_{t_k}^{t_k+\eta_k} p_{\nu_k}^2(t)\,dt \leq \eta\,Np_{\nu_k}. \tag{37}$$

Thus: $\quad \eta\,Np_{\nu_k}>\varepsilon^2 \quad$ or $\quad Np_{\nu_k}>\varepsilon^2/\eta$

which stands in contradiction with the admission (31), since η may be selected as a no matter how little number.

Thus the lemma is proved.

Let us pass over to the proof of the theorem. From its condition we have the relation of the completeness:

$$\lim_{n\to\infty}\int_a^b\left(f'(t)-\sum_{\nu=1}^{n} c_\nu\,\varphi_\nu(t)\right)^2 dt=0 \tag{38}$$

where $c_\nu=\int_a^b f'(t)\,\varphi_\nu(t)\,dt$. On the score of the lemma from (38) follows:

$$\left[f(t)-\sum_{\nu=1}^{n} c_\nu\,\psi_\nu(t)\right] \Rightarrow 0 \tag{39}$$

whereof we get:

$$f(t)=\sum_{\nu=1}^{\infty} c_\nu\,\psi_\nu(t) \tag{40}$$

whereby the series in the right hand side converges uniformly. Thus the theorem is proved. Consequently the system $\psi_\nu(t)$ is a complete system of functions. If in particular we have a system of orthogonal polynomes, the conjunction of their integrals will also form a complete system of functions in regard to any differentiable function, which in case of $t=a$ can be transformed into zero. However one can show that the system of functions $\psi_\nu(t)$ is a complete one in regard to any continuous function. In fact let $\Phi(t)$ be the given continuous function. Then one can select such a second continuous function $\Psi(t)$, that $\Psi(a)=0$ and

$$\int_a^b[\Phi(t)-\Psi(t)]^2\,dt<\varepsilon \tag{41}$$

where ε is a no matter how little positive number[2]).

According to *Weierstrass'* theorem one can select such a polynome $L(t)$, that

$$\Psi(t)-L(t)<\varepsilon \tag{42}$$

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1) In this case we shall make use of the terminology adopted by *R. Courant* and *Hilbert* in »Methoden der Mathematischen Physik« 1924.

2) As an example of the function $\Psi(t)$ the following function may serve:

$$\Psi(t)=(t-a)/l\cdot\Phi(a+l) \quad (a\leq t\leq a+l) \qquad \Psi(t)=\Phi(t) \quad (a+l\leq t\leq b)$$

where $l=\varepsilon/4M^2$. In this case M represents the upper limit $|\Phi(t)|$.

in the whole interval (a, b). Thus

$$\int_a^b [\Psi(t)-L(t)]^2\,dt<(b-a)\,\varepsilon^2\,. \qquad (43)$$

Since $\Psi(a)=0$, $L(a)<\varepsilon$. If we denote $M(t)=L(t)-L(a)$ we can apply to $M(t)$ the formula (39), i. e. we can select such a n that

$$\int_a^b \Big(M(t)-\sum_{\nu=1}^{n} c_\nu\,\psi_\nu(t)\Big)^2 dt<\varepsilon \qquad (44)$$

but from the formulas (41), (42), (43) and (44) there will follow the general relation of the completeness:

$$\lim_{n\to\infty}\int_a^b \Big(\Phi(t)-\sum_{\nu=1}^{n} c_\nu\,\psi_\nu(t)\Big)^2 dt=0 \qquad (45)$$

what we were about to prove.

Whereof already it is readily calculated that the set of functions:

$$\lambda, \lambda^2, \lambda^3, \cdots \lambda^n, \cdots \qquad (46)$$

forms a complete system[1]). If we orthogonalise and normalise this system of functions in the interval from -1 to $+1$, the functions with an odd index will contain merely odd powers, and functions with an even index merely even powers[2]). Therefore, as it is easy to convince oneself, in the interval from 0 to 1 the system of functions:

$$\lambda^2, \lambda^4, \lambda^6, \cdots \lambda^{2n}, \cdots \qquad (47)$$

will be a complete one.

5. It is left to form from the system of functions:

$$\lambda^2, \lambda^4, \lambda^6, \cdots \lambda^{2n}, \cdots$$

a system of orthogonal polynomes normalised in the interval (0,1). Here we indicate the first four polynomes, we have calculated:

$$\left.\begin{aligned}
Q_1(\lambda)&=\sqrt{5}\,\lambda^2\\
Q_2(\lambda)&=10.5\,\lambda^4-7.5\,\lambda^2\\
Q_3(\lambda)&=15.575\,\lambda^2-56.067\,\lambda^4+44.050\,\lambda^6\\
Q_4(\lambda)&=184.252\,\lambda^8-331.654\,\lambda^6+178.583\,\lambda^4-27.058\,\lambda^2
\end{aligned}\right\}(48)$$

In order to develop the function $\psi(\lambda)$ we shall have to calculate the Fourier coefficients of the same function by means of a numeric integration. Therefore it is indispensable to have the tables of the polynomes $Q_1(\lambda)$, $Q_2(\lambda)$, $\cdots$ through equal intervals. The table of the first four polynomes which we have calculated follows with the paper. An active assistance at the calculations of this table was lent to us by *D. I. Eropkin* and *G. N. Fabrikant*, we take the opportunity to express here our most sincere thanks.

Making use of the formula (23) we can calculate the coefficients in the polynomes P_1, P_2, P_3, P_4, into which the function $K(s)$ may be developed, which polynomes correspond to the first four polynomes of the development of the function ψ.

Thus in the first approximation we have:

$$K(s)=du/ds=c_1P_1(s)+c_2P_2(s)+c_3P_3(s)+c_4P_4(s)\,. \qquad (49)$$

Since $u=Ei_3\tau$, by means of the integration (49) we obtain:

$$Ei_3\tau=c_1R_1(s)+c_2R_2(s)+c_3R_3(s)+c_4R_4(s)+C \qquad (50)$$

where

$$R_i(s)=\int_0^s P_i(s)\,ds\,. \qquad (51)$$

We may admit in the formula (50) the permanent quantity to be equal to zero, since in case of $\tau\to\infty$ we have $\lim Ei_3\tau=0$; on the other hand one can practically esteem that in this case also $T\to\infty$, and consequently $s\to 0$, whereof follows that $C=0$. The polynomes $R_1(s)$, $R_2(s)$, $R_3(s)$, $R_4(s)$ are calculated once for all and finally we obtain for $Ei_3\tau$:

$$Ei_3\tau=c_1R_1(s)+c_2R_2(s)+c_3R_3(s)+c_4R_4(s) \qquad (52)$$

where

$$\left.\begin{aligned}
R_1(s)&=0.6797\,s^2\\
R_2(s)&=0.4042\,s^4-2.2797\,s^2\\
R_3(s)&=0.06014\,s^6-2.1584\,s^4+4.7342\,s^2\\
R_4(s)&=0.00455\,s^8-0.4528\,s^6+6.8750\,s^4-8.2246\,s^2
\end{aligned}\right\}(53)$$

6. Before we pass over to practical considerations concerning the question of the numeric solution of the integral equation (14), at first let us consider the question of the soleness of its solution. First of all let us prove that the kernel $[e^{s/\lambda}-1]^{-1}$ is a general one. Since our equation is extended over an infinite district, we shall determine the generality somehow differently from the way first adopted by *Hilbert*[3]).

Determination: The kernel $K(\lambda, s)$ being determined for the whole district $0\leq\lambda\leq\infty$, $0\leq s\leq\infty$ is considered to be a general one, if for any function $a(\lambda)$ one can find such a function $b(s)$ that the following difference

$$x(\lambda)=a(\lambda)-\int_0^\infty K(\lambda, s)\,b(s)\,ds \qquad (A)$$

can be submitted to the condition

$$\int_0^\infty e^{-\lambda^2}\,x(\lambda)^2\,d\lambda<\varepsilon \qquad (B)$$

where ε represents any positive number proposed in advance.

First of all let us show that for any continuous function [we have in view such a function that $\int_0^\infty e^{-\lambda^2}A(\lambda)^2\,d\lambda$ does exist] one can find such a polynome $M(\lambda)$, which does not contain merely free members of zero power, that:

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1) In this particular case, it is easy to convince oneself in the correctness of the above mentioned also without application of the general theorem proved above. Namely, as we proved above, for any continuous function $\Phi(t)$ one can find such a polynome $M(t)$ that:

$$\int_0^1 [\Phi(t)-M(t)]^2\,dt$$

will be less than any positive number. Since in this case $a=0$, thus $M(0)=0$ and consequently $M(t)$ does not contain any zero power.

2) Whereby we suppose that the orthogonalisation is to be effectuated in the ordinary course. Concerning the question of how the process of orthogonalisation and normalisation is effectuated see *Courant* and *Hilbert*, loc. cit. p. 34. 3) *Hilbert, D.* Integralgleichungen, p. 25.

$$\int_0^\infty e^{-\lambda^2}[A(\lambda)-M(\lambda)]^2\,d\lambda<\varepsilon \tag{54}$$

no matter how the positive number ε may be. First let us select such a function $B(\lambda)$ which transforms itself into zero in case of $\lambda=0$, and satisfies the following condition:

$$\int_0^\infty e^{-\lambda^2}[A(\lambda)-B(\lambda)]^2\,d\lambda<\eta \tag{55}$$

where η represents any little positive number. This is readily done, as it is easy to convince oneself (compare note to **4**). We may extend the function $B(\lambda)$ also over the district $-\infty<\lambda\leqq 0$, if we attribute to the same function the oddness. Thus (as one can see from *Sturm-Liouville*'s problem referred to *Hermite*'s polynomes) one can find the following linear combination of a great enough and finite number of polynomes:

$$\alpha_1 H_1(\lambda)+\alpha_2 H_2(\lambda)+\alpha_3 H_3(\lambda)+\cdots+\alpha_n H_n(\lambda) \tag{56}$$

that

$$\int_{-\infty}^{+\infty} e^{-\lambda^2}\left(B(\lambda)-\sum_{i=1}^{n}\alpha_i H_i(\lambda)\right)^2 d\lambda<\eta_2 \tag{57}$$

where η_2 represents a no matter how little positive number, and that

$$\alpha_1 H_1(0)+\alpha_2 H_2(0)+\alpha_3 H_3(0)+\cdots+\alpha_n H_n(0)=\varkappa<\eta_3 \tag{58}$$

where η_3 is also no matter how little. One can easily see that by way of a conformable selection of the numbers η_2 and η_3 one can obtain

$$\int_{-\infty}^{+\infty} e^{-\lambda^2}\left(B(\lambda)-\sum_{i=1}^{n}\alpha_i H_i(\lambda)-\varkappa\right)^2 d\lambda<\eta_4 \tag{59}$$

where η_4 is as well a proposed positive number. From (55), (57), (58) and (59) follows (54).

If we orthogonalise and normalise the system of functions:

$$\lambda,\ \lambda^2,\ \lambda^3,\ \cdots \lambda^n,\ \cdots \tag{60}$$

the same system, being complete in the sense of (54), will be divided into two parts, as each of the reciprocally orthogonal polynomes will contain merely powers of similar evenness. Whereof the completeness of the system of functions in the interval $(0,\infty)$ is readily calculated:

$$\lambda^2,\ \lambda^4,\ \lambda^6,\ \cdots \lambda^{2n},\cdots. \tag{61}$$

Thus one always can find such a polynome $\mathfrak{A}(\lambda)$, containing merely even powers, to begin with the second one, that

$$\int_0^\infty e^{-\lambda^2}[a(\lambda)-\mathfrak{A}(\lambda)]^2\,d\lambda<\varepsilon \tag{62}$$

no matter how the positive number ε may be.

Still from **3** follows that for the polynome $\mathfrak{A}(\lambda)$ one can find such a polynome $b(s)$ that:

$$\mathfrak{A}(\lambda)=\int_0^\infty [e^{s/\lambda}-1]^{-1}\,b(s)\,ds\,. \tag{63}$$

If we denote:

$$x(\lambda)=a(\lambda)-\mathfrak{A}(\lambda) \tag{64}$$

then

$$x(\lambda)=a(\lambda)-\int_0^\infty [e^{s/\lambda}-1]^{-1}\,b(s)\,ds \tag{65}$$

whereby the condition (B) is taken in consideration. Thus the generality of our kernel is proved. Now let us prove that our kernel is secluded from the left. Let us assume the contrary, i. e. let us admit the existence of such a function $q(\lambda)$ that in any case of s:

$$\int_0^\infty [e^{s/\lambda}-1]^{-1}\,q(\lambda)\,e^{-\lambda^2}\,d\lambda=0\,. \tag{66}$$

Whereby:

$$\int_0^\infty e^{-\lambda^2}q^2(\lambda)\,d\lambda=1\,. \tag{67}$$

If so, no matter how the function $v(s)$ may be, for the function

$$x(\lambda)=q(\lambda)-\int_0^\infty [e^{s/\lambda}-1]^{-1}\,v(s)\,ds \tag{68}$$

we shall have:

$$\int_0^\infty e^{-\lambda^2}x(\lambda)^2\,d\lambda=\int_0^\infty [q(\lambda)-\bar{v}(\lambda)]^2\,e^{-\lambda^2}\,d\lambda$$

wherein is introduced the notation

$$\bar{v}(\lambda)=\int_0^\infty [e^{s/\lambda}-1]^{-1}\,v(s)\,ds\,. \tag{69}$$

Since from (66) follows:

$$\int_0^\infty e^{-\lambda^2}q(\lambda)\,\bar{v}(\lambda)\,d\lambda=0$$

thus

$$\int_0^\infty e^{-\lambda^2}x(\lambda)^2\,d\lambda=\int_0^\infty e^{-\lambda^2}q(\lambda)^2\,d\lambda+\int_0^\infty e^{-\lambda^2}\bar{v}^2(\lambda)\,d\lambda>1 \tag{70}$$

which stands in contradiction to the generality of the kernel. Thus the seclusion from the left is proved. Now let us admit that our kernel is not secluded from the right. In this case such a function $p(s)$ must exist which satisfies the following condition

$$\int_0^\infty [e^{s/\lambda}-1]^{-1}\,p(s)\,ds=0 \tag{71}$$

in every case of λ, and moreover the condition

$$\int_0^\infty e^{-s^2}p^2(s)\,ds=1\,. \tag{72}$$

Substituting $1/\lambda=u$, $1/s=t$ we find

$$\int_0^\infty [e^{u/t}-1]^{-1}\,p(1/t)\cdot 1/t^2\cdot dt=0 \tag{73}$$

which stands in contradiction to the seclusion from the left as the formulas (73) and (66) have a similar sense.

From the seclusion from the right follows the soleness of the solution of the equation (14), since, if we admit the existence of two solutions different from each other, the existence of a solution of equation (71) should follow, which should be different from zero.

7. On the score of the above investigations we see, that the problem of a numeric solution of the integral equation (14) comes to the statement of the coefficients: c_1, c_2, $c_3\cdots$.

Since the polynomes (48) are orthogonal and normalised, they may be calculated according to *Fourrier*'s formulas:

$$c_1 = \int_0^1 \psi(\lambda)\, Q_1(\lambda)\, d\lambda \qquad c_3 = \int_0^1 \psi(\lambda)\, Q_3(\lambda)\, d\lambda$$
$$c_2 = \int_0^1 \psi(\lambda)\, Q_2(\lambda)\, d\lambda \qquad c_4 = \int_0^1 \psi(\lambda)\, Q_4(\lambda)\, d\lambda\,. \tag{75}$$

Having in view that the function $\psi(\lambda)$ has been obtained from observations, the calculation of the coefficients c_1, c_2, c_3, c_4 comes to a numeric integration.

How completely and exactly, in every particular case, the development of the function $\psi(\lambda)$ into polynomes $Q(\lambda)$ has been fulfilled one can judge by the preciseness observed in the relation of completeness:

$$\int_0^1 \psi(\lambda)^2\, d\lambda = \sum c_i^2\,. \tag{75}$$

In any case *Bessel*'s inequality must be taken in consideration:

$$\sum_{i=1}^{4} c_i^2 \leqq \int_0^1 \psi(\lambda)^2\, d\lambda\,. \tag{76}$$

We must notice that from observations made we obtain $\varphi(\lambda)$ [and consequently also $\psi(\lambda)$] with the exactness of a constant multiplicator. If we denote by $\chi(\lambda)$ the visible distribution of energy in the spectrum of a star, we shall have:

$$\chi(\lambda) = \pi R^2/r^2 \cdot \varphi(\lambda) \tag{77}$$

where r is the distance from the star. Consequently if $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ are the coefficients of the development of the function $\lambda^5 \chi(\lambda)/2c_1$ into polynomes Q_1, Q_2, Q_3, Q_4 thus

$$\gamma_i = \pi R^2/r^2 \cdot c_i \tag{78}$$

and the formula (52) may be expressed as follows:

$$\pi R^2/r^2 \cdot Ei_3\tau = \gamma_1 R_1(s) + \gamma_2 R_2(s) + \gamma_3 R_3(s) + \gamma_4 R_4(s) \tag{79}$$

Since we can calculate for any star the coefficients $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ consequently for any star we can obtain the following relations:

$$\pi R^2/r^2 \cdot Ei_3\tau = \beta_1 (c_2/T)^2 + \beta_2 (c_2/T)^4 + \beta_3 (c_2/T)^6 + \beta_4 (c_2/T)^8 \tag{80}$$

where

$$\left.\begin{aligned} \beta_1 &= 0.6797\,\gamma_1 - 2.2797\,\gamma_2 + 4.7342\,\gamma_3 - 8.2246\,\gamma_4 \\ \beta_2 &= 0.4042\,\gamma_2 - 2.1584\,\gamma_3 + 6.8750\,\gamma_4 \\ \beta_3 &= 0.06014\,\gamma_3 - 0.4528\,\gamma_4 \\ \beta_4 &= 0.00455\,\gamma_4 \end{aligned}\right\} \tag{81}$$

as one easily can see from the formulas (79) and (53). If we admit in the formula (80) $T = T_0$ the temperature of the upper limit, we obtain, since $Ei_3\,0 = \frac{1}{2}$

$$\pi R^2/2r^2 = \beta_1 (c_2/T_0)^2 + \beta_2 (c_2/T_0)^4 + \beta_3 (c_2/T_0)^6 + \beta_4 (c_2/T_0)^8 \tag{82}$$

i. e. we obtain the relation between the radius and the temperature of the upper limit. Since we know T_0 we can find the graphic of the dependence between τ and T, making use of the formula (80) and as well of the tables of the function $Ei_3\tau$, quoted below[1]).

8. To illustrate the exposed method we show below its application to the following three stars: γ Draconis, α Andromedae and α Ceti. The distribution of the brightness in the spectra of these stars has been taken according to the measurements effectuated by *J. Wilsing*, *J. Scheiner* and *W. Münch*[2]) in the Observatorium in Potsdam. For the above stars they have measured the brightness in ten various lengths of waves, namely:

0.451 μ 0.472 μ 0.494 μ 0.514 μ 0.535 μ 0.556 μ 0.577 μ 0.593 μ 0.615 μ 0.642 μ.

In virtue of these points we have composed a graphic of the function $\chi(\lambda)$ in some arbitrary scale (for the value of the function). As for the scale of the length of waves, it has been thus selected, that its unit should contain 0.666 μ. Thus the argument λ alters from 0 to 1, this being indispensable since the polynomes, into which the function $\chi(\lambda)$ may be developed, are orthogonalised and normalised from 0 to 1. The observations effectuated by *Wilsing*, *Scheiner* and *Münch* do not determine the value of the function $\chi(\lambda)$ for an alteration of the argument from 0.00 to 0.67 (in the new scale), thus we had to join in some arbitrary way the point of the curve, the abscissa of which is $\lambda = 0.676$, with the origin of the coordinates. However this arbitrariness is almost hardly perceptible in the results, as, the multiplicator λ^5 being present, the values of the function $\chi(\lambda)$ in the indicated interval are small and, the numeric integration being effectuated, this arbitrariness does not give any perceptible fault. The numeric calculation of the integrals (74) has been divided into two parts. We have integrated the function $\chi(\lambda)\, Q_i(\lambda)$ separately in the interval from 0.0 to 0.6 and separately in the interval from 0.6 to 1.0. These calculations have been effectuated according to *Simpson*'s formula:

$$\int_a^b f(x)\, dx =$$
$$(b-a)/6n \cdot [y_0 + 2y_1 + 4y_2 + 2y_3 + \cdots + 4y_{2n-2} + 2y_{2n-1} + y_{2n}]. \tag{83}$$

In the interval from 0.0 to 0.6 we have taken $n = 3$, and in the interval from 0.6 to 1.0 $n = 10$. The multiplication of the values of the functions $\chi(\lambda)$ by the values of the polynomes $Q_i(\lambda)$ has been effectuated by means of an arithmometer and also the addition according to formula (83). As result of these operations were obtained the coefficients $\gamma_1, \gamma_2, \gamma_3, \gamma_4$ whereof by means of formula (81) the equation (80) was obtained for each of the three investigated stars. Below we give a table of the coefficients of this equation for the above stars:

Star	Spectral-type	β_1	β_2	β_3	β_4
γ Draconis	K5	+0.07459	−0.07226	+0.00637	−0.00004
α Andromedae	A	+0.05796	−0.08071	+0.00264	−0.00002
α Ceti	Ma	+0.0558	−0.0813	+0.0079	−0.0001

[1]) The table containing $2Ei_3\tau$ is composed according to the formula

$$2Ei_3\tau = e^{-\tau}(1-\tau) + \tau^2 Ei\tau$$

where the value of the function $Ei\tau$ has been taken from: *Jahnke* und *Emde*, Funktionentafeln, 2. Aufl.

[2]) *J. Wilsing*; Effective Temperaturen von 199 hellen Sternen nach spektralphotometrischen Messungen von *J. Wilsing*, *J. Scheiner* und *W. Münch*, Publ. d. Astrophys. Observ. zu Potsdam, Nr. 74, p. 66–73.

9. Now the main difficulty consists in $\pi R^2/r^2$ or, what would be the same thing, in the ignorance of T_0. In the first approximation we have taken $T_0 = T_{\text{eff.}}/\sqrt[4]{2}$. Thus the permanent multiplicator for $Ei_3\tau$ has been determined, hereafter according to equation (80), a graphic of the dependence between T^4 and τ has been composed. If we denote $A(\tau) = T^4$ and $A_0 = T_0{}^4$, for γ Draconis we have the equation:

$$A(\tau) = A_0 + 0.90 \cdot 10^{14}\tau + 0.27 \cdot 10^{14}\tau^2 \tag{84}$$

for α Ceti:

$$A(\tau) = A_0 + 0.87 \cdot 10^{14}\tau + 0.15 \cdot 10^{14}\tau^2 \tag{85}$$

and for α Andromedae:

$$A(\tau) = A_0 + 0.86 \cdot 10^{16}\tau + 0.39 \cdot 10^{16}\tau^2\,. \tag{86}$$

To make a comparison we mention the same equation for the sun:

$$A(\tau) = A_0 + 0.71 \cdot 10^{15}\tau\,. \tag{87}$$

Thus we can state that the gradient of radiation of an absolutely black body, per every unit of the optical mass, is ninefold greater in the sun than the same gradient in the stars, which belong to later spectral types. But this gradient for α Andromedae is more than tenfold greater than the gradient for the sun and has the tendency to increase, when we proceede into the inner layers.

Thus on the evidence of the above mentioned, yet scanty facts we can say that the gradient $dA/d\tau$ increases with the transition from later spectral types to earlier ones. However the solving of the problem of the influence of the arbitrariness in the selection of T_0 on the values of the coefficients in the formulas (84), (85) and (86) is of great interest. For this purpose some calculations relating to the star γ Draconis have been effectuated which proved that by alteration of T_0 by 200° this coefficient does not alter perceptibly as the alteration is confined in the limits of the preciseness of the graphic.

Having for each star the following equation:

$$A(\tau) = A_0 + A_1\tau + A_2\tau^2 \tag{88}$$

we can find, according to *Schwarzschild*, the distribution of the brightness over the disk of the star. Namely if we denote by x the cosine of the angular distance of the point from the centre of the disk, we shall have:

$$I(x) = A_0 + A_1 x + 2A_2 x^2\,. \tag{89}$$

Thus the brightness $I(x)$ on the disks of γ Draconis, α Ceti and α Andromedae is a square function of the cosine of the angular distance from the centre of the disk, while for the sun it is a linear one.

It seems to us that the application of our method to stars of the Algol-type may contribute to precisise the elements which are calculated for their orbits, as the distribution of the brightness over their disks will be known to us. Let us notice that as we know $A(\tau)$ and consequently also $B(\tau)$ we can calculate by means of formula (1) $f(\tau)$, i. e. the quantity of formed energy as a function of the optical mass. Moreover, making use of the equation which expresses the mechanic equilibrium of a star, having adopted the equation of the state of ideal gases and knowing $B(\tau)$ we can investigate also the gradient of the temperature for the unit of the length, if we form some hypothesis relating to the molecular weight.

1. Tables of the Polynomes $Q_1(s)$, $Q_2(s)$, $Q_3(s)$, $Q_4(s)$.

s	$Q_1(s)$	$Q_2(s)$	$Q_3(s)$	$Q_4(s)$
0.000	0.000	0.000	0.000	0.000
0.005	0.000	0.000	0.000	−0.001
0.010	0.000	−0.001	+0.002	0.003
0.015	+0.001	0.002	0.003	0.006
0.020	0.001	0.003	0.006	0.011
0.025	0.001	0.005	0.010	0.017
0.030	+0.002	−0.007	+0.014	−0.024
0.035	0.003	0.009	0.019	0.033
0.040	0.004	0.012	0.025	0.043
0.045	0.005	0.015	0.031	0.055
0.050	0.006	0.019	0.039	0.067
0.055	+0.007	−0.022	+0.047	−0.080
0.060	0.008	0.027	0.055	0.096
0.065	0.009	0.031	0.065	0.111
0.070	0.011	0.036	0.075	0.129
0.075	0.013	0.042	0.086	0.147
0.080	+0.014	−0.048	+0.098	−0.166
0.085	0.016	0.053	0.109	0.186
0.090	0.018	0.060	0.122	0.207
0.095	0.020	0.067	0.135	0.230
0.100	0.022	0.074	0.150	0.253
0.105	+0.025	−0.081	+0.165	−0.276
0.110	0.027	0.089	0.180	0.301
0.115	0.030	0.097	0.196	0.327
0.120	0.032	0.106	0.212	0.352
0.125	0.035	0.114	0.230	0.380
0.130	+0.038	−0.124	+0.247	−0.405
0.135	0.041	0.133	0.265	0.434
0.140	0.044	0.143	0.284	0.466
0.145	0.047	0.153	0.303	0.493
0.150	0.050	0.163	0.322	0.521
0.155	+0.054	−0.174	+0.342	−0.550
0.160	0.057	0.185	0.362	0.581
0.165	0.061	0.196	0.382	0.611
0.170	0.065	0.208	0.404	0.642
0.175	0.068	0.220	0.426	0.671
0.180	+0.072	−0.232	+0.447	−0.699
0.185	0.077	0.244	0.469	0.730
0.190	0.081	0.257	0.491	0.761
0.195	0.085	0.270	0.513	0.786
0.200	0.089	0.283	0.536	0.816
0.205	+0.094	−0.296	+0.559	−0.846
0.210	0.099	0.310	0.582	0.875
0.215	0.103	0.324	0.604	0.900
0.220	0.108	0.338	0.627	0.927
0.225	0.113	0.353	0.650	0.954
0.230	+0.118	−0.367	+0.673	−0.979
0.235	0.123	0.382	0.696	1.004
0.240	0.129	0.397	0.719	1.027
0.245	0.134	0.412	0.742	1.049
0.250	0.140	0.428	0.765	1.070
0.255	+0.145	−0.443	+0.788	−1.091
0.260	0.151	0.459	0.810	1.112
0.265	0.157	0.475	0.831	1.130
0.270	0.163	0.491	0.854	1.147
0.275	0.169	0.507	0.876	1.162
0.280	+0.175	−0.524	+0.898	−1.177
0.285	0.182	0.540	0.919	1.190
0.290	0.188	0.556	0.939	1.201
0.295	0.195	0.573	0.960	1.210
0.300	0.201	0.590	0.980	1.218
0.305	+0.208	−0.607	+0.999	−1.225
0.310	0.215	0.624	1.018	1.229
0.315	0.222	0.641	1.036	1.232
0.320	0.229	0.658	1.054	1.232
0.325	0.236	0.675	1.072	1.233
0.330	+0.244	−0.692	+1.088	−1.231
0.335	0.251	0.709	1.104	1.226
0.340	0.258	0.727	1.119	1.220
0.345	0.266	0.744	1.134	1.214
0.350	0.274	0.761	1.148	1.202
0.355	+0.282	−0.778	+1.161	−1.191
0.360	0.290	0.796	1.173	1.178
0.365	0.298	0.813	1.184	1.162
0.370	0.306	0.830	1.194	1.145
0.375	0.314	0.847	1.203	1.125

s	$Q_1(s)$	$Q_2(s)$	$Q_3(s)$	$Q_4(s)$	s	$Q_1(s)$	$Q_2(s)$	$Q_3(s)$	$Q_4(s)$	s	$Q_1(s)$	$Q_2(s)$	$Q_3(s)$	$Q_4(s)$
0.380	+0.323	−0.864	+1.212	−1.103	0.605	+0.818	−1.338	+0.349	+1.065	0.830	+1.540	−0.184	−1.477	−0.820
0.385	0.331	0.881	1.220	1.080	0.610	0.832	1.337	0.302	1.104	0.835	1.559	0.125	1.466	0.921
0.390	0.340	0.898	1.227	1.054	0.615	0.846	1.335	0.253	1.139	0.840	1.578	0.064	1.446	1.019
0.395	0.349	0.914	1.232	1.026	0.620	0.860	1.332	0.204	1.171	0.845	1.597	−0.002	1.428	1.117
0.400	0.358	0.931	1.237	0.996	0.625	0.874	1.328	0.154	1.202	0.850	1.616	+0.062	1.401	1.204
0.405	+0.367	−0.948	+1.241	−0.963	0.630	+0.888	−1.323	+0.104	+1.231	0.855	+1.635	+0.128	−1.363	−1.290
0.410	0.376	0.964	1.243	0.930	0.635	0.902	1.317	+0.052	1.253	0.860	1.654	0.197	1.329	1.371
0.415	0.385	0.980	1.244	0.896	0.640	0.916	1.310	0.000	1.274	0.865	1.673	0.266	1.283	1.444
0.420	0.394	0.996	1.244	0.858	0.645	0.930	1.303	−0.053	1.291	0.870	1.693	0.339	1.230	1.512
0.425	0.404	1.012	1.244	0.818	0.650	0.945	1.294	0.106	1.304	0.875	1.712	0.413	1.171	1.568
0.430	+0.413	−1.028	+1.242	−0.778	0.655	+0.959	−1.285	−0.160	+1.315	0.880	+1.732	+0.489	−1.105	−1.615
0.435	0.423	1.043	1.238	0.738	0.660	0.974	1.275	0.214	1.322	0.885	1.751	0.567	1.029	1.654
0.440	0.433	1.058	1.234	0.695	0.665	0.989	1.263	0.267	1.323	0.890	1.771	0.647	0.949	1.679
0.445	0.443	1.073	1.228	0.649	0.670	1.004	1.251	0.322	1.321	0.895	1.791	0.729	0.858	1.691
0.450	0.453	1.088	1.221	0.599	0.675	1.019	1.238	0.377	1.315	0.900	1.811	0.814	0.760	1.688
0.455	+0.463	−1.103	+1.212	−0.550	0.680	+1.034	−1.223	−0.431	+1.305	0.905	+1.831	+0.900	−0.653	−1.670
0.460	0.473	1.117	1.202	0.501	0.685	1.049	1.207	0.485	1.291	0.910	1.852	0.990	0.537	1.636
0.465	0.484	1.131	1.191	0.450	0.690	1.065	1.191	0.540	1.272	0.915	1.872	1.080	0.409	1.580
0.470	0.494	1.144	1.179	0.399	0.695	1.080	1.173	0.593	1.241	0.920	1.893	1.174	0.273	1.507
0.475	0.505	1.158	1.166	0.345	0.700	1.096	1.154	0.647	1.216	0.925	1.913	1.270	0.128	1.407
0.480	+0.515	−1.171	+1.151	−0.292	0.705	+1.111	−1.134	−0.701	+1.190	0.930	+1.934	+1.368	+0.033	−1.287
0.485	0.526	1.183	1.135	0.235	0.710	1.127	1.113	0.754	1.156	0.935	1.955	1.468	0.197	1.139
0.490	0.537	1.195	1.117	0.180	0.715	1.143	1.090	0.806	1.113	0.940	1.976	1.571	0.378	0.965
0.495	0.548	1.207	1.098	0.123	0.720	1.159	1.066	0.856	1.069	0.945	1.997	1.676	0.567	0.759
0.500	0.559	1.219	1.078	0.065	0.725	1.175	1.041	0.906	1.020	0.950	2.018	1.784	0.777	0.521
0.505	+0.570	−1.230	+1.056	−0.008	0.730	+1.192	−1.015	−0.956	+0.966	0.955	+2.039	+1.890	+0.986	−0.252
0.510	0.582	1.240	1.033	+0.050	0.735	1.208	0.987	1.004	0.915	0.960	2.061	2.006	1.214	+0.055
0.515	0.593	1.250	1.008	0.110	0.740	1.224	0.958	1.050	0.842	0.965	2.082	2.121	1.448	0.400
0.520	0.605	1.260	0.983	0.170	0.745	1.241	0.928	1.095	0.775	0.970	2.104	2.239	1.712	0.787
0.525	0.616	1.269	0.956	0.226	0.750	1.258	0.896	1.139	0.703	0.975	2.126	2.359	1.981	1.218
0.530	+0.628	−1.278	+0.927	+0.288	0.755	+1.275	−0.864	−1.181	+0.629	0.980	+2.148	+2.482	+2.266	+1.695
0.535	0.640	1.286	0.897	0.347	0.760	1.292	0.829	1.221	0.548	0.985	2.170	2.607	2.566	2.220
0.540	0.652	1.294	0.866	0.405	0.765	1.309	0.793	1.258	0.466	0.990	2.192	2.738	2.880	2.798
0.545	0.664	1.301	0.834	0.460	0.770	1.326	0.757	1.294	0.379	0.995	2.214	2.866	3.210	3.432
0.550	0.676	1.308	0.799	0.520	0.775	1.343	0.717	1.327	0.289	1.000	2.236	3.000	3.548	4.123
0.555	+0.689	−1.314	+0.764	+0.577	0.780	+1.360	−0.676	−1.356	+0.196					
0.560	0.701	1.319	0.729	0.630	0.785	1.378	0.635	1.378	0.095					
0.565	0.714	1.324	0.692	0.685	0.790	1.396	0.591	1.410	+0.002					
0.570	0.727	1.328	0.652	0.737	0.795	1.413	0.546	1.431	−0.098					
0.575	0.739	1.332	0.612	0.789	0.800	1.431	0.499	1.449	0.197					
0.580	+0.752	−1.335	+0.571	+0.840	0.805	+1.449	−0.451	−1.465	−0.302					
0.585	0.765	1.337	0.529	0.891	0.810	1.467	0.401	1.475	0.405					
0.590	0.778	1.338	0.485	0.936	0.815	1.485	0.349	1.482	0.510					
0.595	0.792	1.339	0.442	0.983	0.820	1.504	0.296	1.485	0.616					
0.600	0.805	1.339	0.395	1.024	0.825	1.522	0.241	1.483	0.718					

2. Table of the function $2Ei_3\tau$.

τ	$2Ei_3\tau$	τ	$2Ei_3\tau$	τ	$2Ei_3\tau$	τ	$2Ei_3\tau$	τ	$2Ei_3\tau$	τ	$2Ei_3\tau$	τ	$2Ei_3\tau$	τ	$2Ei_3\tau$
0.00	1.0000	0.06	0.8936	0.20	0.7039	0.50	0.4432	0.80	0.2886	1.40	0.1291	2.60	0.0289	5.00	0.0018
0.01	0.9806	0.07	0.8777	0.25	0.6494	0.55	0.4119	0.85	0.2693	1.60	0.0998	2.80	0.0227	6.00	0.0006
0.02	0.9619	0.08	0.8622	0.30	0.6001	0.60	0.3831	0.90	0.2513	1.80	0.0774	3.00	0.0178		
0.03	0.9440	0.09	0.8472	0.35	0.5553	0.65	0.3566	0.95	0.2348	2.00	0.0603	3.20	0.0141		
0.04	0.9267	0.10	0.8325	0.40	0.5146	0.70	0.3321	1.00	0.2194	2.20	0.0471	3.40	0.0111		
0.05	0.9098	0.15	0.7645	0.45	0.4773	0.75	0.3095	1.20	0.1679	2.40	0.0368	4.00	0.0055		

Leningrad, 1927 Oct.

N. A. Kosirev, V. A. Ambarzumian.

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Ausgemessene photographische Positionen Kleiner Planeten. Von *K. Reinmuth*.

Planet	Datum	M. Z. Kgst.	α	δ	Äquin.	Anschlußsterne	Bm.
1927 CB	1927 Febr. 7	$13^h48^m_.3$	$9^h33^m46^s_.79$	+19° 55′ 40″.6	1927.0	Berl A 3868, Berl B 3814	1
»	1927 Febr. 8	14 20.1	9 33 11.43	+19 58 40.4	»	» »	
»	1927 März 4	10 29.4	9 16 23.51	+21 18 34.0	»	Berl B 3722, 3733+EB.	
»	1027 März 24	10 26.0	9 7 59.65	+21 43 7.4	»	Berl B 3658, 3698	
»	1927 März 28	10 53.4	9 7 13.50	+21 43 6.7	»	» »	
1927 CE	1927 Febr. 8	14 20.1	9 30 58.46	+18 20 26.7	»	Berl A 3846, Kü 4251	1
1927 CF	»	»	9 33 0.07	+17 37 5.5	»	Berl A 3866, 3879	1
1927 CG	»	»	9 38 22.18	+21 2 51.3	»	Kü 4279, Berl B 3828	1
933 [1927 CH]	1927 Febr. 9	15 26.7	10 19 16.05	+11 30 53.9	»	Lpz I 4007, 4010+EB.	2, 3
»	1927 Febr. 23	12 8.6	10 8 1.77	+13 39 41.5	»	Kü 4498, Lpz I 3970	
»	1927 März 21	8 35.4	9 52 17.89	+16 40 9.4	»	Berl A 3977, 3989	
1927 CJ	1927 Febr. 10	16 33.7	11 47 29.37	+ 2 18 17.8	»	Alb 4356, 4371	4
1927 CK	»	»	11 47 55.11	+ 4 40 12.9	»	Alb 4359, Lpz II 5948	4
1927 CL	»	»	11 52 17.69	+ 4 44 36.9	»	Lpz II 5948, Kü 5299	4, 1
1927 CM	»	»	11 55 2.84	+ 2 43 48.0	»	Alb 4383, 4395	
1927 CN	»	»	11 55 34.01	+ 2 45 32.6	»	» »	
1927 CO	»	»	12 1 4.04	+ 3 11 19.7	»	Alb 4414, 4419+EB.	
»	1927 März 4	13 21.8	11 48 48.77	+ 5 0 7.2	»	Lpz II 5934, 5950	5
1927 DB	1927 Febr. 23	12 8.6	10 18 0.07	+ 8 36 29.4	»	Lpz II 5442, 5467	5
1927 EA	1927 März 3	10 18.5	9 21 59.19	+17 53 54.7	»	Berl A 3793+EB., 3820	1
»	1927 März 4	10 29.4	9 21 15.30	+17 52 30.7	»	Berl A 3796, 3801	
»	1027 März 24	10 26.0	9 9 55.40	+17 6 6.2	»	Berl A 3727, Kü 4096	
»	1927 März 28	10 53.4	9 9 19.43	+16 52 21.5	»	Berl A 3719, 3734	6
1927 FA	1927 März 21	8 35.4	10 2 19.98	+14 1 2.7	»	Lpz I 3932, 3948	5
1927 HA	1927 April 22	10 58.8	14 10 36.66	− 3 35 52.9	»	Strb 5053, 5070	
1927 JA	1927 Mai 3	11 0.7	14 32 45.79	− 5 22 2.7	»	Strb 5164, 5173+EB.	7
1927 LA	1927 Juni 1	11 46.3	16 35 10.51	+ 1 34 16.7	»	Alb 5495, 5528	
1927 MA	1927 Juni 29	12 18.5	19 38 14.11	− 5 6 1.1	»	Strb 6753+EB., 6780	
»	1927 Juli 4	12 23.7	19 33 56.98	− 5 8 23.3	»	Strb 6711, 6744	1
1927 OA	1927 Juli 30	11 39.2	22 23 46.06	+11 59 22.5	»	Kü 9942, Lpz I 8972	
1927 PA	1927 Aug. 3	13 12.6	22 59 36.37	−10 48 20.4	»	* *a*, * *b*	8
1927 QA	1927 Aug. 29	10 55.5	23 14 34.07	− 1 37 5.9	»	Strb 8017+EB., Nic 5808	3
1927 QB	»	14 20.5	23 30 7.65	− 6 18 40.9	»	Ott 8339, 8352	6
1927 QC	1927 Aug. 30	10 51.0	0 13 29.86	+13 9 47.0	»	Lpz I 53+EB., 78	
»	1927 Sept. 28	10 34.1	23 54 15.49	+13 16 27.7	»	Lpz I 9501, 9514	
?1927 QC	1927 Okt. 16	8 13.6	23 41 37.88	+11 49 31.0	»	Lpz I 9422, 9426	9, 1
1927 QD	1927 Aug. 30	10 51.0	0 15 21.14	+15 37 48.7	»	Berl A 56+EB., Lpz I 85	
»	1927 Sept. 28	10 34.1	23 52 45.14	+15 27 21.6	»	Berl A 9734+EB., Lpz I 9499	5
»	1927 Okt. 16	8 13.6	23 39 22.60	+14 27 52.6	»	Lpz I 9403, Kü 10511	
1927 QE	1927 Aug. 30	10 51.0	0 23 0.10	+16 19 18.7	»	Kü 144+EB., Berl A 112+EB.	10, 7
»	1927 Sept. 28	10 34.1	0 2 1.76	+15 45 10.0	»	Lpz I 9540, Kü 8	
»	1927 Okt. 16	8 13.6	23 48 53.65	+14 18 14.7	»	Lpz I 9458, 9483	5
1927 QF	1927 Aug. 30	14 21.0	0 36 42.45	− 0 2 29.2	»	Nic 102+EB., 114	
»	1927 Sept. 26	11 35.1	0 19 2.54	− 1 27 17.4	»	Nic 45, Strb 81	
»	1927 Okt. 26	9 38.6	23 58 33.26	− 2 37 53.6	»	Strb 8186, 1+EB.	
1927 QG	1927 Aug. 30	14 21.0	0 39 46.99	+ 0 26 26.9	»	Nic 115+EB., 126	
1927 QH	»	»	0 45 48.17	− 1 29 7.7	»	Strb 176, Nic 148+EB.	
»	1927 Sept. 26	11 35.1	0 30 15.77	− 4 7 23.0	»	* *c*, * *d*	
»	1927 Okt. 26	9 38.6	0 11 30.55	− 6 23 18.6	»	Ott 33, 46	
»	1927 Okt. 27	9 28.0	0 11 3.94	− 6 25 54.5	»	» »	